Learning Goals

Most physical phenomena are described by **more than one** dependent variables. In class we have just encountered the case of **parametrized curves** where to describe the position of an object, as a function of time, we need to simulaneously examine its x-coordinate and y-coordinate as a function of time.

Today we will explore parametrized curves, and start you thinking about how to apply the calculus concepts of **integrals** and **derivatives** to these curves, ahead of the rigorous discussion we will give in class. We will start our discussion with the help of the Etch A Sketch toy, and finish by analyzing the vehicle data from a 1963 Corvette Grand Sport on the Laguna Seca race track.

Etch a Sketch

The Etch a Sketch is a toy invented in the 1960s.

Figure 1: The Etch A Sketch toy, with a drawing of Taj Mahal. Image source `https://commons.wikimedia.org/wiki/File:Taj_Mahal_drawing_on_an_Etch-A-Sketch.jpg` used under CC-by-SA license.

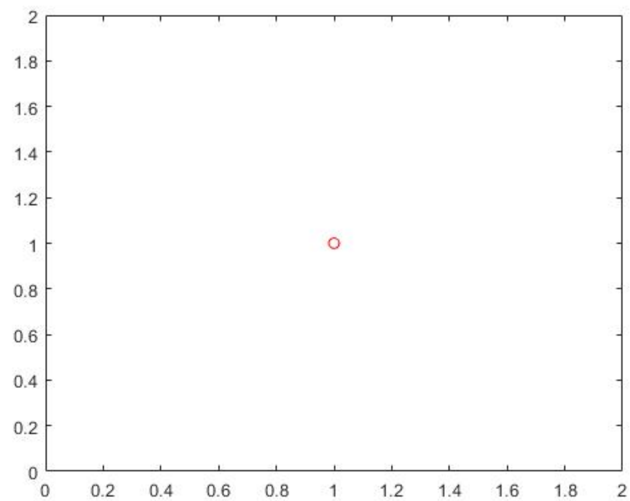
The toy has two knobs. Turning the left one moves a tracer up and down, and turning the right one moves a tracer left and right. Mathematically, this means that the **vertical rate of change** of the tracer is given by the rate at which you twist the left knob, and the **horizontal rate of change** of the tracer is given by the rate at which you twist the right knob.

The following graphic illustration will walk you through a stimulation of the Etch a Sketch toy using some simple `MATLAB` commands to prepare you for more complicated examples in the lab.

Left knob

Let imagine the tracer starting at the point $(1,1)$, to plot this on MATLAB, we would use this line of code

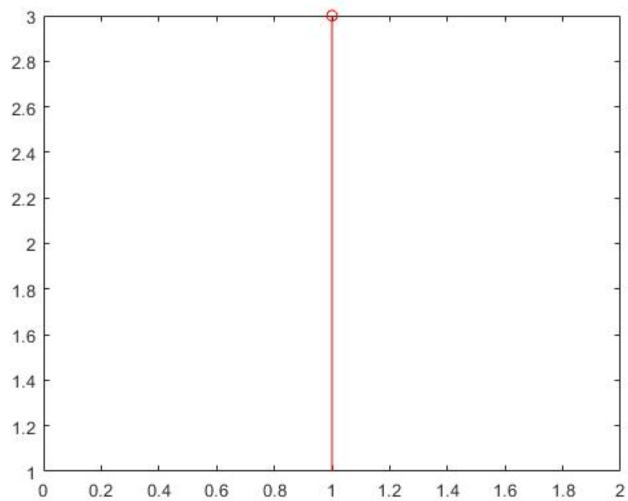
```
plot(1,1,'ro')
```



and as you can see, the tracer, a red circle, is now located at the point $(1,1)$.

Now we will turn the left knob steadily at 2 units per second. We started at the point $(1,1)$, but after turning the knob, the tracer moves up 2 units, so it ends at the point $(1,3)$. We plot the line and the tracer with the following code

```
plot([1,1], [1,3], 'r')
plot(1, 3, 'ro')
```

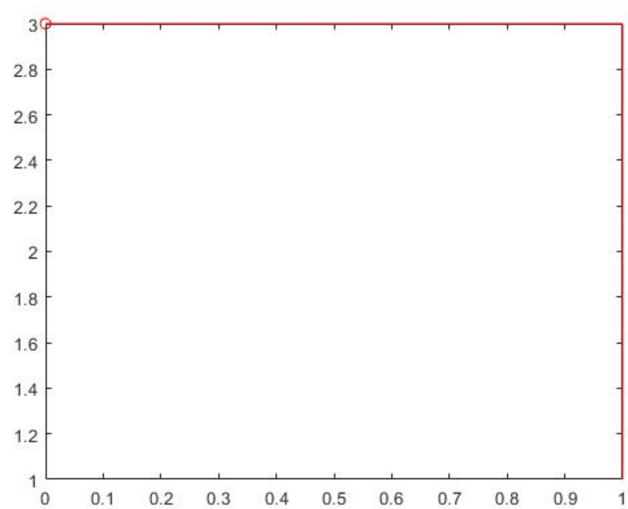


The plot command first takes the list of x values and then the list of y values.

Right knob

We now turn the right knob steadily at -1 unit per second, for a length of 1 second. At the end of the last step, the tracer was at the point $(1, 3)$. Next we turned the right knob, so the tracer moved to the left by 1 unit, ending at the point $(0, 3)$. We record all of our movement so far in the following line of code producing the corresponding trace on the Etch a Sketch.

```
plot([1,1,0], [1,3,3], 'r')
plot(0, 3, 'ro')
```

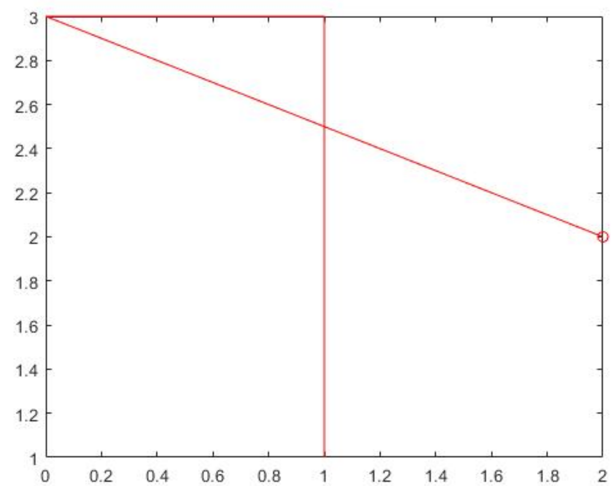


Two knobs together

We now turn the left knob and the right knob together, with the left knob going at -1 units per second, and the right knob moving at 2 units per second, for a duration of 1 second.

The left knob controls the y-coordinate, so moving at -1 units per second for 1 second will decrease the y-coordinate by 1. Starting from the point $(0, 3)$, it will move to a point with y-coordinate 2. As the right knob controls the x-coordinate, moving it at 2 units per second for 1 second will increase the x-coordinate by 2. Starting from $(0, 3)$, it will move to a point with x-coordinate 2. To trace the path with the final coordinate as $(2, 2)$, we use the following code.

```
plot([1,1,0,2],[1,3,3,2],'r')
plot(2, 2, 'ro')
```



How far has the tracer travelled?

There are three steps to this process.

- In the first step, the tracer moved up by 2 units.
- In the second step, the tracer moved sideways by 1 unit.
- In the final step, the tracer moved diagonally. It moved 2 units right and 1 unit down. The distance travelled in this step can be found using the Pythogorean theorem: the distance travelled is the length of the hypotenuse. So,

$$\text{distance} = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{1^2 + 2^2} = \sqrt{1 + 4} = \sqrt{5}$$

So in total, the distance travelled by the tracer is $2 + 1 + \sqrt{5} = 3 + \sqrt{5}$ units.